Time-domain finite-element homogenisation of laminated iron cores with net circulating currents

Johan Gyselinck[1], Patrick Dular[2], Laurent Krähenbühl[3], Ruth V. Sabariego[4]

[1] BEAMS Department, Université Libre de Bruxelles (ULB), Belgium, johan.gyselinck@ulb.ac.be
[2] ACE, Dept. of Electrical Engineering and Computer Science, University of Lie'ge (ULg), Belgium, patrick.dular@ulg.ac.be
[3] Université de Lyon, Ampere (CNRS UMR5005, Ecole Centrale de Lyon, INSA-Lyon, UCBL), France
[4] Dept. Electrical Engineering (ESAT), EnergyVille, KU Leuven, Belgium, ruth.sabariego@esat.kuleuven.be

This paper deals with the time-domain homogenization of laminated cores in 2D or 3D finite element (FE) models of electromagnetic devices, allowing for net circulating current in the laminations (e.g. due to imperfect or damaged insulation). The homogenization is based on the expansion of the induction through the lamination thickness in terms of orthogonal polynomial basis functions, in conjunction with a magnetic vector potential (MVP) formulation. The even skin-effect basis functions are linked to the net flux. We add now the odd ones to account for the net current. The approach is validated through a linear 2D test case. The extension to 3D nonlinear problems is straightforward.

Index Terms—Eddy currents, finite-element methods, homogenization, lamination stack.

I. INTRODUCTION

WHEN MODELLING real-life electromagnetic devices comprising laminated iron cores by means of the FE method, it is unfeasible to discretise each lamination separately. Dedicated numerical techniques are required to precisely and efficiently account for the induced eddy currents, the associated losses and the ensuing skin effect. The approaches proposed in literature so far are applicable in frequency and/or time domain, consider saturation (possibly hysteresis) or not, include perpendicularly incident flux, and consist of single-step or two-step algorithms [1]-[4]. Mostly perfect insulation of the laminations is assumed, i.e. the induced current density cancels in any cross-section of a lamination. In practice net circulating current may occur due to inter-lamination insulation damage [5].

In this paper the net-current feature is added to the nonlinear time-domain homogenization method proposed in [4]. The procedure is similar to the thin-shell technique in [6]. For the sake of brevity the test case in this digest is linear and 2D.

II. 1D LAMINATION MODEL

Let us consider a lamination of thickness d ($-d/2 \leq z \leq d/2$), of homogeneous isotropic material having a constant conductivity σ (resistivity $\rho = \sigma^{-1}$) and constant permeability μ (reluctivity $\nu = \mu^{-1}$). The magnetic induction $b(z,t)$ and magnetic field $h(z,t) = \nu\, b(z,t)$ are assumed along e.g. the x-axis, and the current density $j(z,t)$ and electric field $e(z,t) = \rho\, j(z,t)$ along the y-axis. The 1D eddy-current problem is governed by

$$\partial_z^2 h(z,t) = \sigma\, \partial_t b(z,t)\,. \tag{1}$$

Regarding the symmetry of $b(z,t)$ and $j(z,t)$ with respect to $z = 0$, two dual cases can be distinguished, as shown in Fig. 1, with net flux and net current in the lamination respectively.

For these two cases, the analytical frequency-domain solution of (1), at frequency f and pulsation $\omega = 2\pi f$, yields
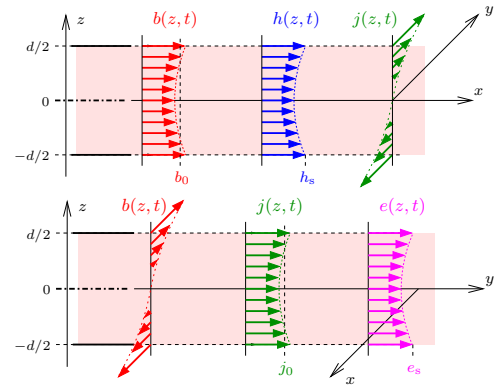

Fig. 1. 1D lamination model with 2 symmetrical cases: net flux with even $b(z,t)$ and $h(z,t)$ and odd $j(z,t)$ (up) versus net current with even $j(z,t)$ and $e(z,t)$ and odd $b(z,t)$ (down)

the following expressions of the complex lamination-level reluctivity $\boldsymbol{\nu}$ and resistivity $\boldsymbol{\rho}$ respectively (complex quantities in bold, $\boldsymbol{\jmath} = \sqrt{-1}$):

$$\boldsymbol{\nu} = \frac{\boldsymbol{h}_\mathrm{s}}{\boldsymbol{b}_0} = \nu\, \boldsymbol{\Gamma}(d^*) \quad \text{and} \quad \boldsymbol{\rho} = \frac{\boldsymbol{e}_\mathrm{s}}{\boldsymbol{\jmath}_0} = \rho\, \boldsymbol{\Gamma}(d^*)\,, \tag{2}$$

with $\boldsymbol{b}_0$ and $\boldsymbol{\jmath}_0$ the average induction and current density resp., $\boldsymbol{h}_\mathrm{s}$ and $\boldsymbol{e}_\mathrm{s}$ the surface magnetic and electric field resp. The frequency dependence is contained in $\boldsymbol{\Gamma}(d^*)$ through the relative lamination thickness $d^* = d/\delta$ (with $\delta = \sqrt{2/(\omega\mu\sigma)}$ the penetration depth):

$$\boldsymbol{\Gamma}(d^*) = \frac{1+\boldsymbol{\jmath}}{2}\, d^* \operatorname{cotanh}\!\left(\frac{1+\boldsymbol{\jmath}}{2}\, d^*\right). \tag{3}$$

An approximate time-domain solution of (1) can be obtained through expansion of $b(z,t)$ with both even and odd polynomial basis functions $\alpha_k(z)$ up to order n:

$$b(z,t) = \sum\nolimits_{k=0}^{n} \alpha_k(z)\, b_k(t)\,. \tag{4}$$

By choosing n, e.g. Legendre polynomials as $\alpha_k(z)$ [4], one may compromise between accuracy and computational cost. With $n = 0$, i.e. neglecting skin effect, low-frequency eddy-current losses are effected, at practically no additional cost. The case $n = 2$ is developed very briefly hereafter. Thanks to $\alpha_2(z)$ and $\alpha_1(z)$ skin effect (of net flux) and net current can be considered respectively.

With a view to the incorporation in the FE equations (in terms of the magnetic vector potential) four differential equations regarding the next flux and net current result:

$$\begin{bmatrix} h_s \\ 0 \end{bmatrix} = \nu \begin{bmatrix} 1 & 0 \\ 0 & 1/5 \end{bmatrix} \begin{bmatrix} b_0 \\ b_2 \end{bmatrix} + \sigma d^2 \begin{bmatrix} 1/12 & -1/60 \\ -1/60 & 1/210 \end{bmatrix} \partial_t \begin{bmatrix} b_0 \\ b_2 \end{bmatrix},$$
(5)

$$j_0 = \frac{2\nu}{d} b_1 + \frac{\sigma d}{30} \partial_t b_1 \quad \text{and} \quad e_s = \frac{2\nu}{d\sigma} b_1 + \frac{d}{5} \partial_t b_1,$$
(6)

with $h_s(t) = (h(d/2, t) + h(-d/2, t))/2$ and $e_s(t) = (e(d/2, t) + e(-d/2, t))/2$.

In the FE model, a coarse mesh can then be used in the homogenized lamination stack, and for each basic unknown herein (MVP values associated to an edge e.g.) there are $n-1$ additional unknowns.

III. Application Example

The presented homogenization approach is applied to the 2D model shown in Fig. 2. It comprises a core of 2×10 laminations with $d = 0.5$ mm, $\sigma = 2\ 10^6$ S/m, and relative permeability $\mu_r = 2000$. The magnetic field and induction are perpendicular to the plane, whereas all current density, imposed and induced, is in the plane of the model.
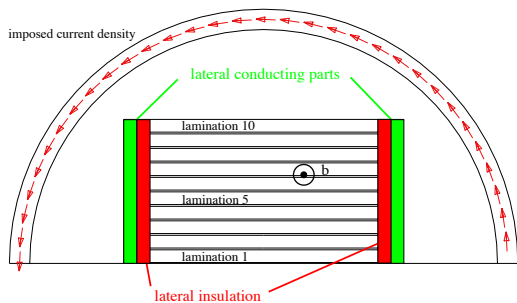

Fig. 2. 2D FE model with laminated core (half cross-section of the ring core with toroidal coil considered in [4] but ignoring curvature)

Net induced currents in the laminations are possible due to the presence of the non-magnetic conducting layers depicted in red and green in Fig. 2. For the lateral insulation layer (in red), we assume either $\sigma_i = 0$ (perfect insulation) or $\sigma_i = 2\ 10^3$ S/m. For the layer in green we assume $\sigma_g = 5\ 10^6$ S/m.

A 1 kHz sinusoidal current is imposed with an amplitude such that the induction in the lamination would be uniform with a 1 T amplitude in absence of all induced currents ($\sigma = 0$, $\sigma_i = 0$ and $\sigma_g = 0$). In presence of induced currents in the laminations only ($\sigma_i = 0$ and $\sigma_g = 0$), there is a clear skin effect inside the lamination as $d^* = d/\delta = 1.99$, with the same induction profile along each lamination thickness. In presence of conducting insulation and lateral parts, a global skin effect appears (see Fig. 3).

The 3D MVP formulation adopted is the same as in [4]. A fine (reference model) and a course mesh (homogenized model) are adopted with resp. twelve and just one layer of quadrangles per lamination (triangles elsewhere). They yield a total of resp. 10000 and 1000 complex unknowns.

With non-conducting layers and the fine mesh, the average induction in each lamination is $\boldsymbol{b}_{av} = (0.697 - \jmath\, 0.398)$ T. With the frequency-domain homogenization, using the coarse mesh and the complex $\boldsymbol{\nu}(d^*)$, we obtain $(0.683 - \jmath\, 0.405)$ T.

With conducting layers and the fine mesh, the average induction in the core is $(0.166 - \jmath\, 0.280)$ T; see Fig. 3. The frequency-domain (FD) homogenization, using $\boldsymbol{\rho}(d^*)$ as well, produces $(0.168 - \jmath\, 0.281)$ T. The time-domain (TD) homogenization with $n = 2$ exhibits (in steady-state and after phasor calculation) $(0.163 - \jmath\, 0.280)$ T; see Fig. 3. The agreement can be improved by increasing n.
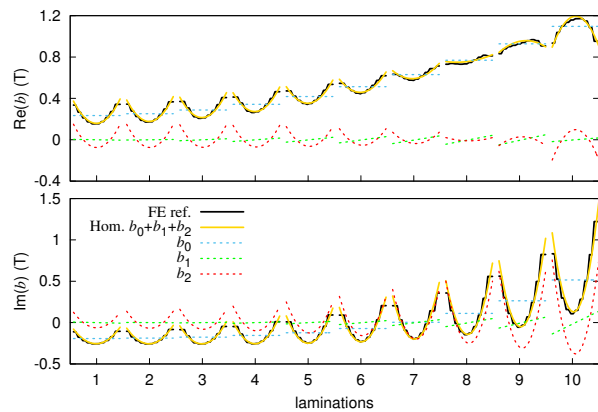

Fig. 3. Variation of induction throughout the ten laminations of the core with imperfect lateral insulation (with imposed real valued current), obtained with fine mesh (black) and with TD homogenization (in yellow), plus components b_0, b_1 and b_2 (in blue, green and red dashed line resp.)

In the extended paper, the proposed method will be elaborated in detail and more results will be given and analyzed.

Acknowledgement

References

[1] K. Hollaus and O. Biro, "A FEM formulation to treat 3D eddy currents in laminations," *IEEE Trans. Magn.,* vol. 36, pp. 1289–1292, 2000.
[2] O. Bottauscio, and M. Chiampi, "Analysis of Laminated Cores Through a Directly Coupled 2-D/1-D Electromagnetic Field Formulation," *IEEE Trans. Magn.,* vol. 38, no. 5, pp. 2358–2361, 2002.
[3] J. Pippuri and A. Arkkio, "Time-harmonic induction-machine model including hysteresis and eddy currents in steel laminations," *IEEE Trans. Magn.,* vol. 45, no. 7, pp. 2981–2989, 2009.
[4] J. Gyselinck, R. V. Sabariego, P. Dular, "A nonlinear time-domain homogenization technique for laminated iron cores in 3-D finite-element models," *IEEE Trans. Magn.,* vol. 42, no. 4, pp. 763–766, 2006.
[5] S. B. Shah, P. Rasilo, A. Belahcen, A. Arkkio, "Modeling of losses due to inter-laminar short-circuit currents in lamination stacks," *Electrical, Control and Communication Engineering,* vol. 3, no. 1, pp. 31–36, 2013.
[6] R. V. Sabariego, C. Geuzaine, P. Dular, J. Gyselinck, "Nonlinear time-domain finite-element modeling of thin electromagnetic shell," *IEEE Trans. Magn.,* vol. 45, no. 3, pp. 976–979, 2009.